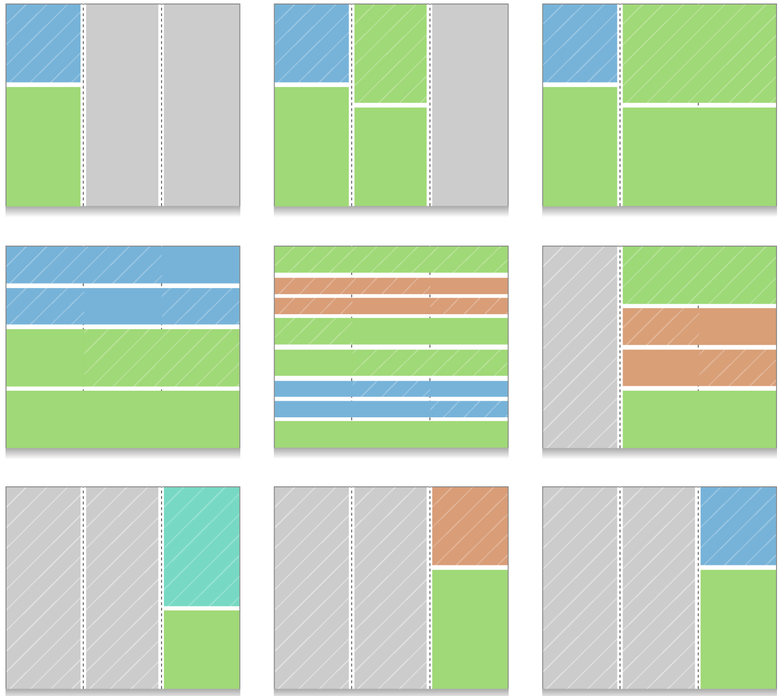
# Dynamic software visualization of quantum algorithms with rainbow boxes

Jean-Baptiste Lamy
jean-baptiste.lamy @ univ-paris13.fr



Université Paris 13, 93017 Bobigny
Sorbonne Universités, Paris
INSERM UMRS 1142

# Introduction

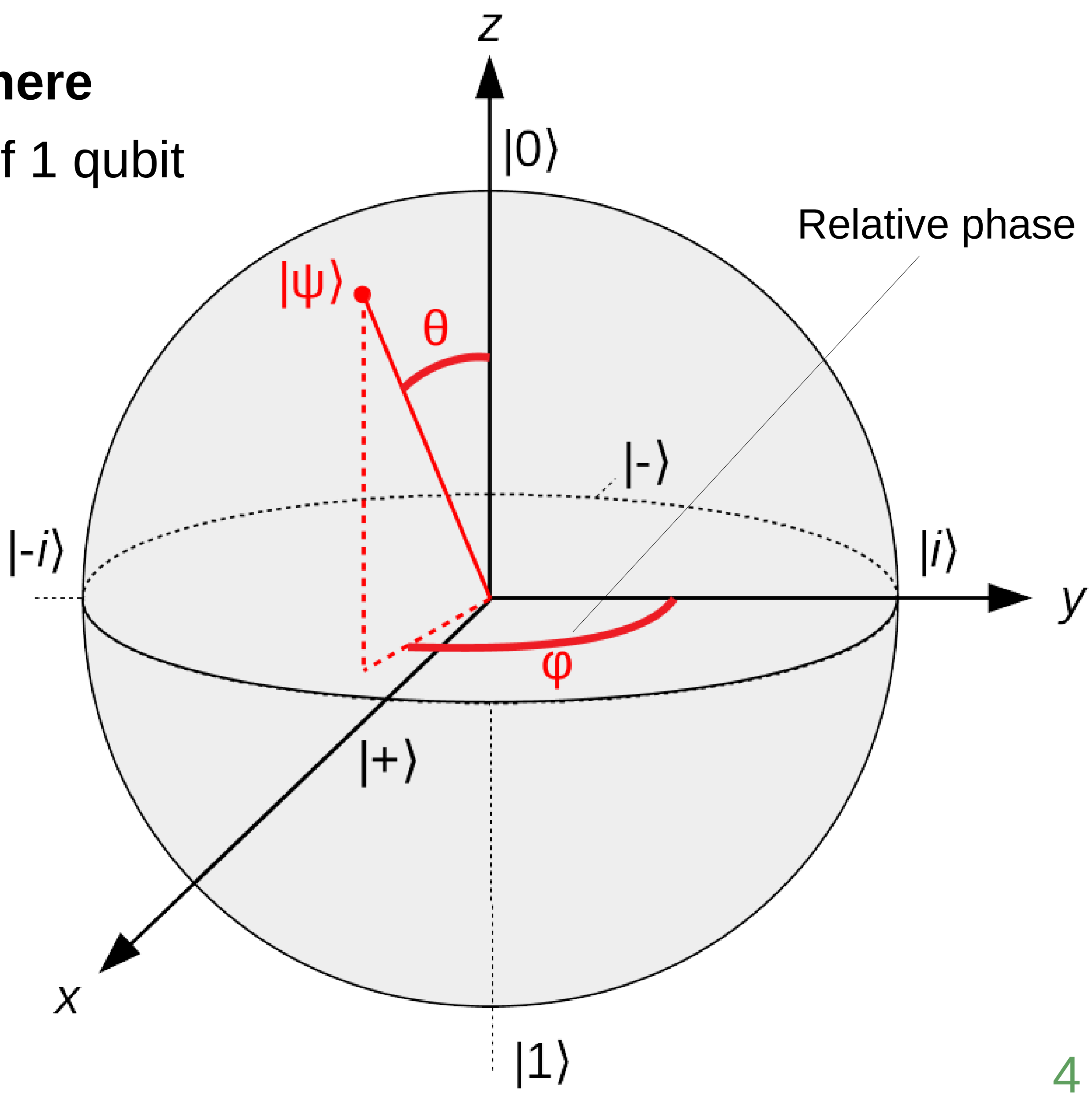
- **Quantum computing**
  - Combines quantum physics, computer science and information theory
  - Can outperform classical algorithms in terms of complexity
  - Requires specific hardware and algorithms
    - Often complex and unintuitive
- **Software visualization**
  - Represents graphically
    - Algorithms, software, source codes (static approach)
    - Runtime data or memory (dynamic approach)
- **Here, we propose a dynamic software visualization approach to quantum algorithms**
  - Visualizes the quantum memory (qubits) during the execution of a quantum algorithm
  - Relies on set visualization

# Introduction: quantum computing

- **Classical bits => quantum bits (qubits)**
  - Two states $|0\rangle$ and $|1\rangle$ (Dirac notation)
  - But also superpositions of these two states: $a|0\rangle + b|1\rangle$
  - A and b are complex numbers with $|a|^2 + |b|^2 = 1$
  - 1 qubit has the computing power of 2 real numbers
- **BUT**
  - When measured, 1 qubit produces **only** 1 classical bit of information (*e.g.* 0 or 1).
    - 0 is obtained with probability $|a|^2$, and 1 with probability $|b|^2$
  - Several distinct superpositions exist with the same probabilities of measuring 0 and 1
    - They differ by their relative phase
    - It has no impact on the measure
    - But can impact other operations performed on the qubit

# Introduction: quantum computing

- **Surface of the Bloch sphere**
  - Represents the state of 1 qubit
  - Cannot be applied to more than one qubit



# Introduction: quantum computing

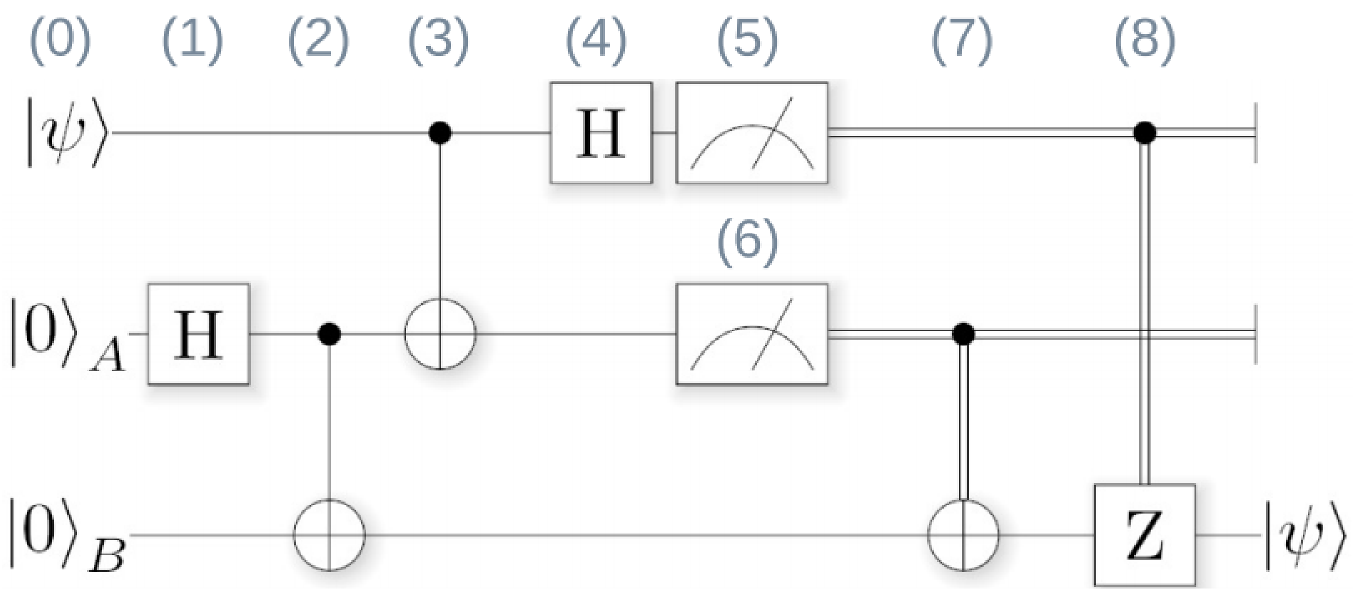
- **Multiple qubits**
  - Due to *quantum entanglement*, the value of the various qubits may not be independent from each other
    - The computing power increases exponentially with the number of qubits
  - The state of n qubits is a superposition of $2^n$ values
    - For 3 qubits (a, b, c… are complex numbers with $|a|^2 + |b|^2 + \ldots = 1$):
      $a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
  - Some states are not fully *entangled* but *separable* in a tensor product
    - For example $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|011\rangle$ can be factored as: $|0\rangle \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle)$
    - When measured, returns 000 or 011 (50%)

qubit #1 is not entangled
qubit #2 and #3 are entangled

  - Relative phases exist also on multiple qubits
    - Several superpositions yield the same probability when measured

# Introduction: quantum circuit

- **Quantum circuit is the main approach to quantum computing**
  - Circuit with quantum gates
  - Used in IBM Q Experience environment for graphical programming
  - Example circuit for quantum teleportation:



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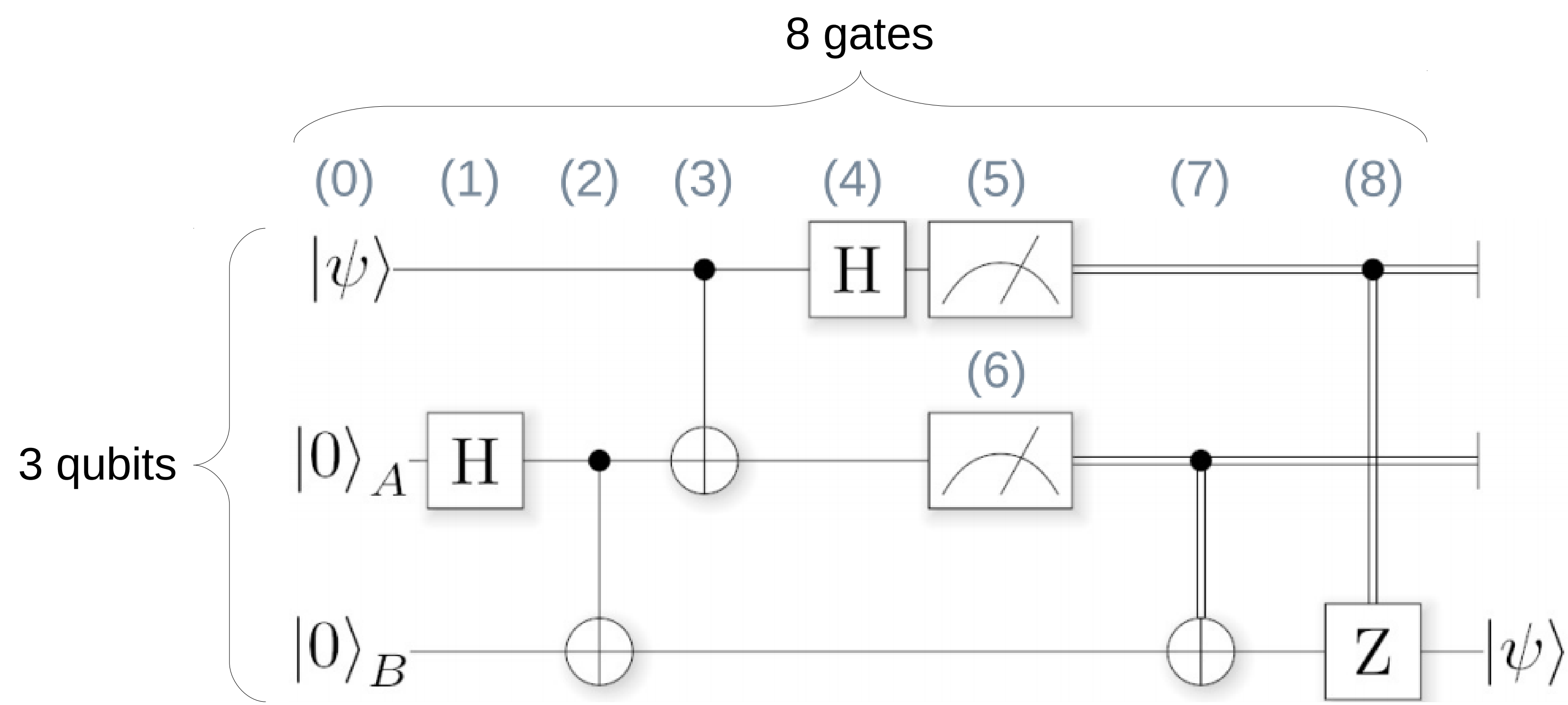

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**Hadamard gate:**
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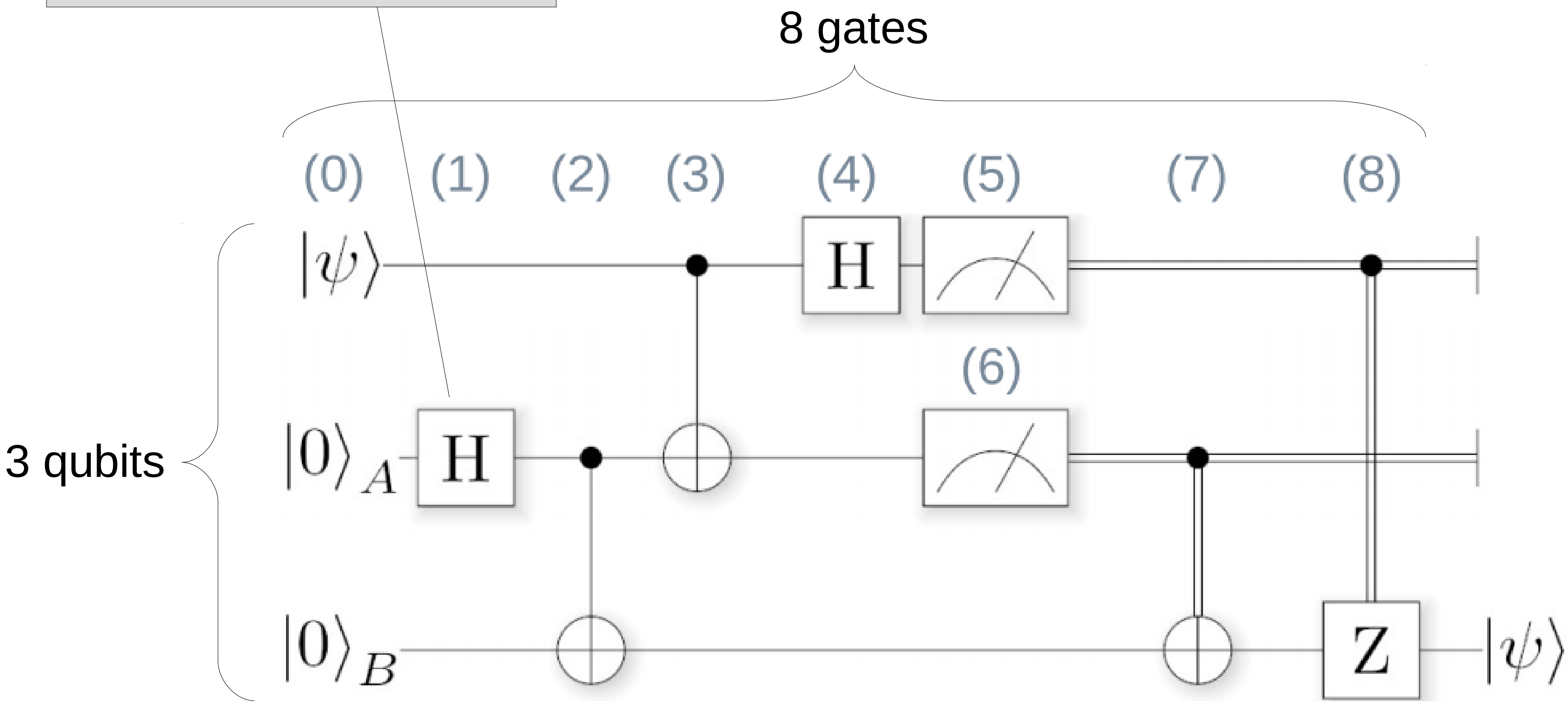
*e.g.* $|0\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$



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if first qubit is $|1\rangle$, swaps
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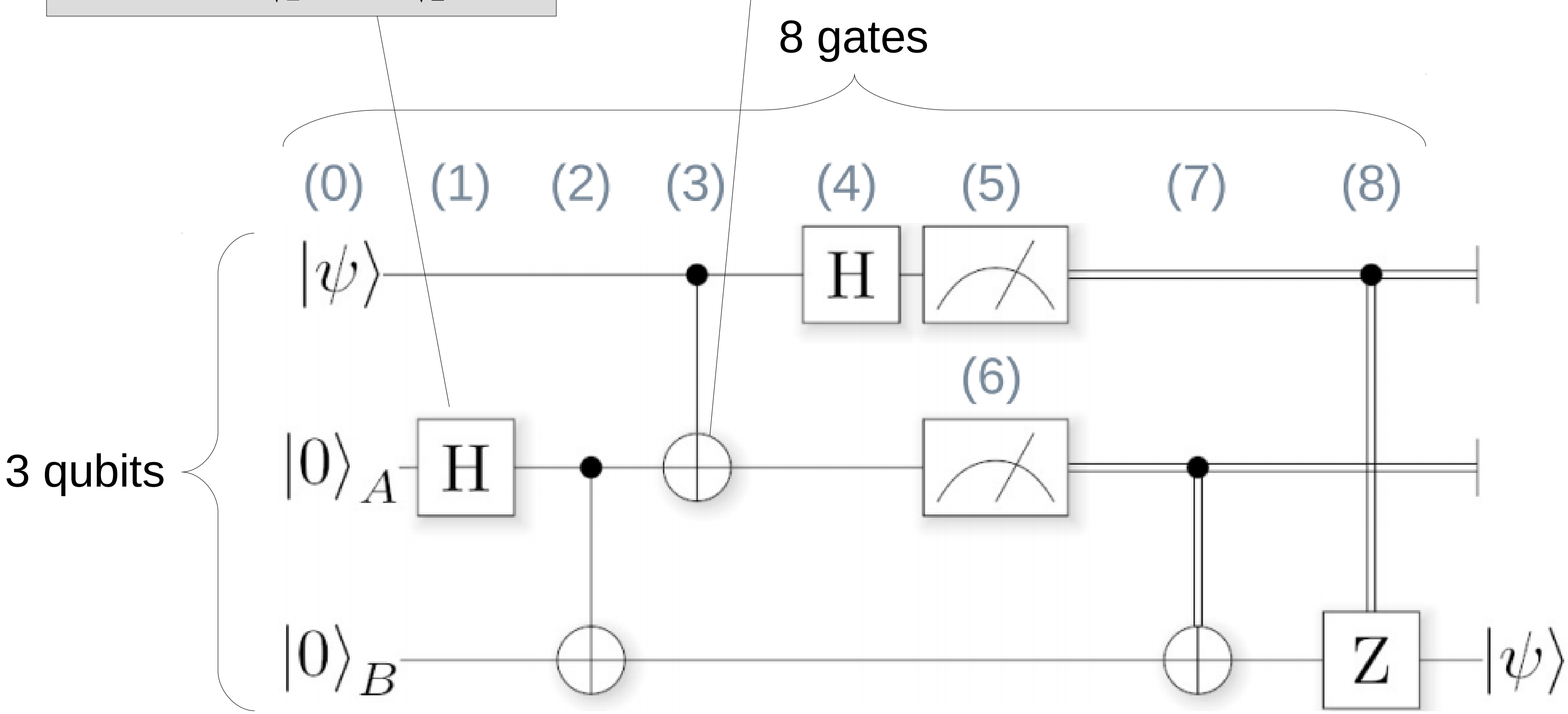
$a|0\rangle + b|1\rangle \rightarrow b|0\rangle + a|1\rangle$



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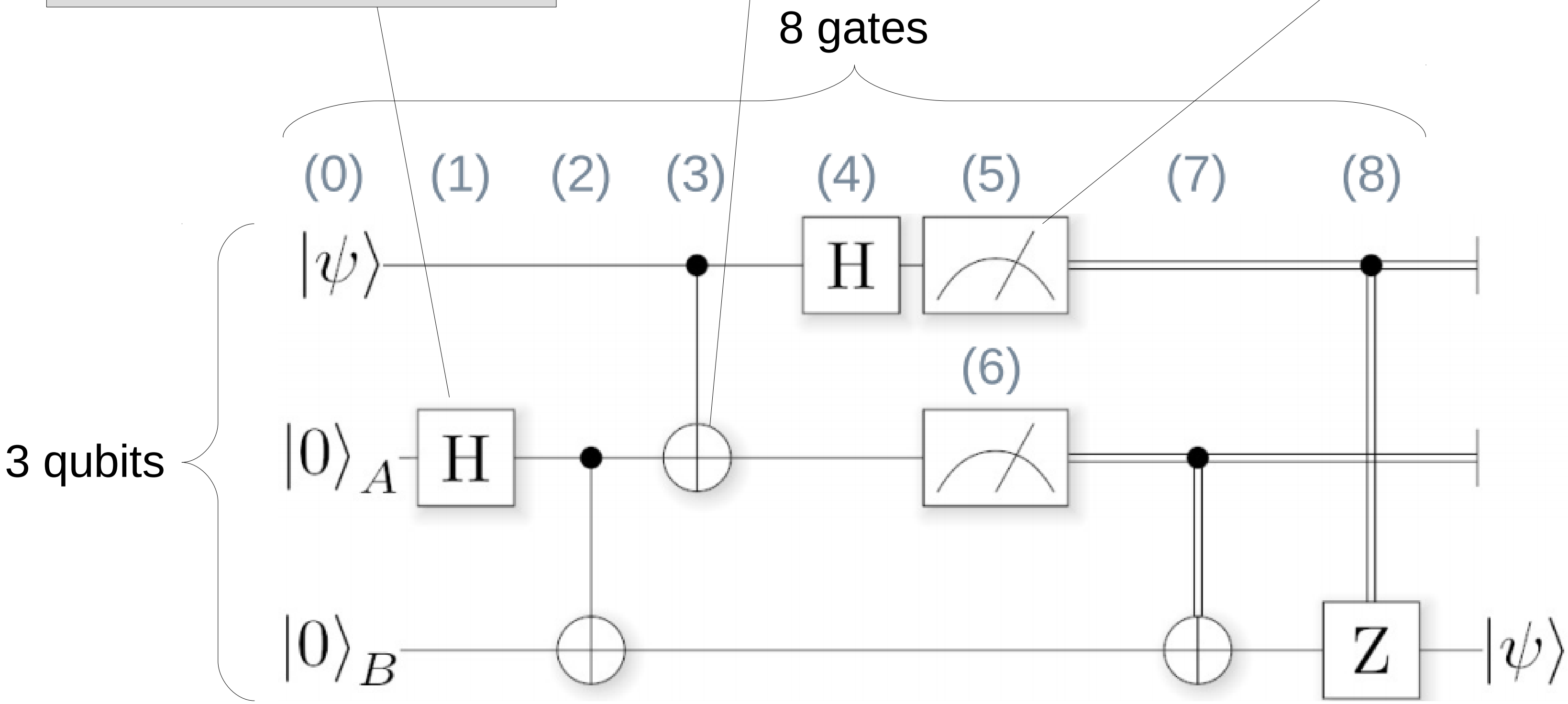
**Measure gate:**
$a|0\rangle + b|0\rangle \rightarrow$ 0 (probability $|a|^2$)
1 (probability $|b|^2$)



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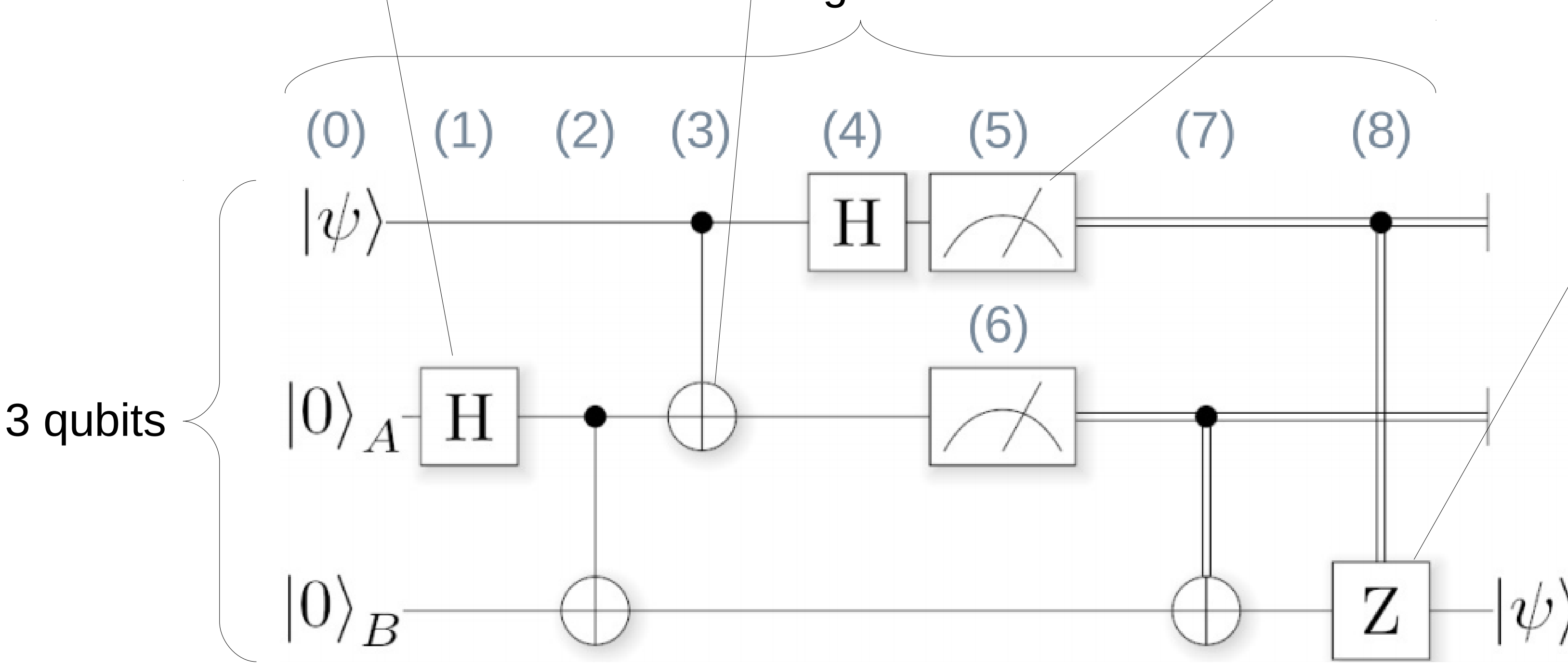
*i.e.* $a|0\rangle + b|1\rangle$
$\rightarrow a|0\rangle - b|1\rangle$



# Introduction: quantum circuit

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  - Example circuit for quantum teleportation:

    Problem: due to entanglement, a gate may modify a qubit not involved in the gate!

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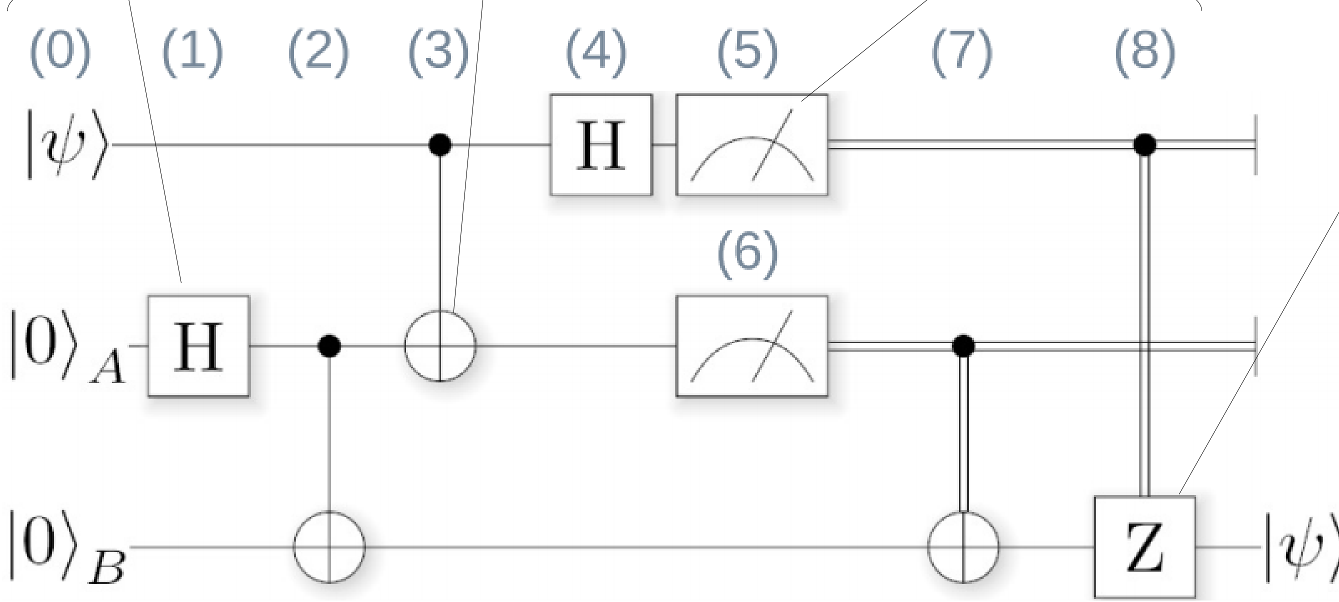
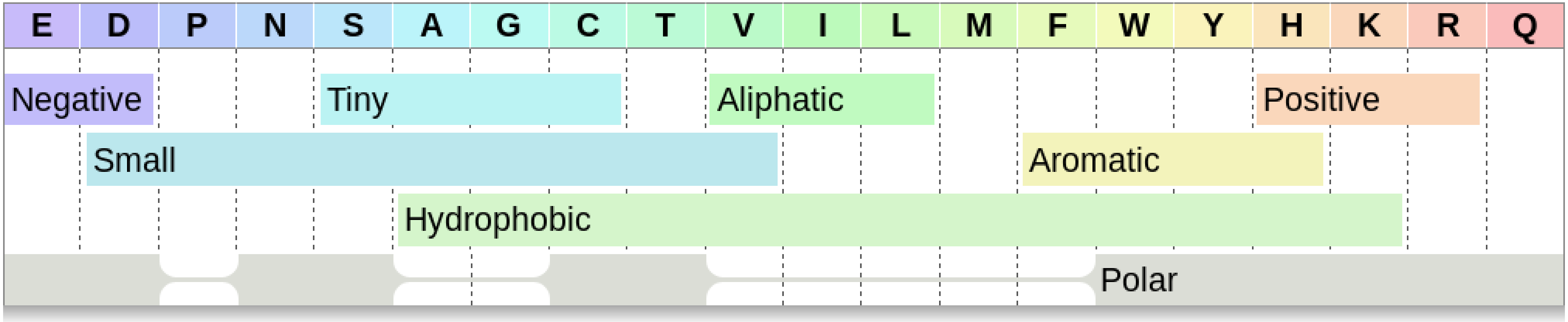
8 gates

3 qubits



# Set visualization




- **Considers sets and elements**

- **Rainbow boxes : a recent technique for set visualization**
  - elements => columns
  - sets => rectangular boxes
  - color => one color per element
  - box color is the mean of its elements color
  - non contiguous element in a set => box hole
  - elements are ordered so as to minimize the number of holes
  - box are stacked vertically by size

**Best paper at iV2017 and iV2018!**

# Unique description of a multiple-qubit state

- **Vector formula (Dirac or matrix notation) are not unique**
  - Due to the global phase phenomenon (only relative phases matter)
  - The two following 2-qubit states are different mathematically, but equivalent physicially (or computationally):

$$\sqrt{\frac{1}{4}}|0\rangle + \sqrt{\frac{3}{4}}i|1\rangle \qquad \left(\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}i\right)|0\rangle + \left(\frac{\sqrt{3}}{2\sqrt{2}}i - \frac{\sqrt{3}}{2\sqrt{2}}\right)|1\rangle$$

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- **We designed a unique representation of multiple-qubit states**
  - **Step 1:** factorize separable states as much as possible

$$\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|011\rangle \longrightarrow |0\rangle \otimes \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$$

  - **Step 2:** describe each superposed states (*e.g.* $|0\rangle$, $|11\rangle$,…) by (p, φ)
    - p is the probability of measuring this state
    - φ is the relative phase

# Unique description of a multiple-qubit state

- Step 1: factorize separable states as much as possible
- Step 2: describe each superposed states (*e.g.* $|000\rangle$, $|011\rangle$,…) by ($p$,$\varphi$)
- **Step 3:** fix a reference phase $\varphi_0$ for each factor
  - $\varphi_0$ is the relative phase of the lowest bit-value state present (e.g. $|00\rangle$)
  - Compute normalized phases $\varphi' = (\varphi - \varphi_0) \bmod 2\pi$

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- **Step 4:** describe uniquely a multiple qubit-state by a set of quadruplets:

$$\{ (q, B, p, \varphi') \}$$

- → q is the subset of qubits involved
- → B is the state
- → p is the probability of measuring the state
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$$|0\rangle \otimes \left( \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle \right) \longrightarrow$$

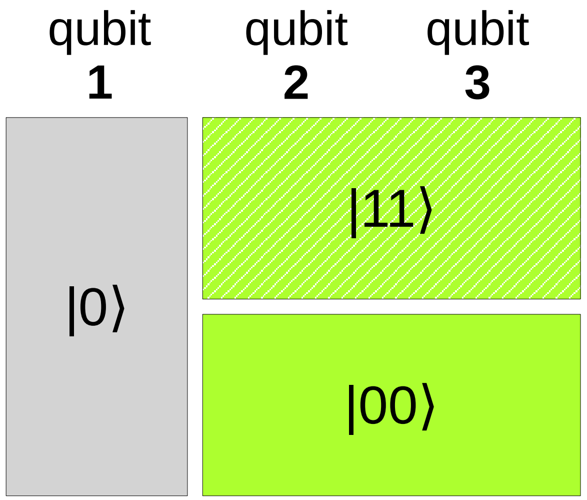
3 quadruplets:
( {1}, $|0\rangle$, 100%, 0 )
( {2, 3}, $|00\rangle$, 50%, 0 )
( {2, 3}, $|11\rangle$, 50%, 0 )

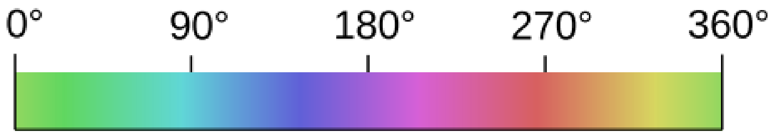
"qubit #1 takes value 0 with probability 100%"
"qubits #2 and #3 take values 00 with probability 50%"
"qubits #2 and #3 take values 11 with probability 50% and a relative phase of 0"

# Visual representation

- **Multiple-qubit state visualization**
  - A typed-set visualization problem (first member of quadruplets)
  - We used rainbow boxes
    - One column per qubit
    - One box per quadruplet
- **Visual encoding:**
  - qubits q: box X position and box width
  - state B: box Y position, hatches, label
  - probability p: box height
  - phase φ’: box color

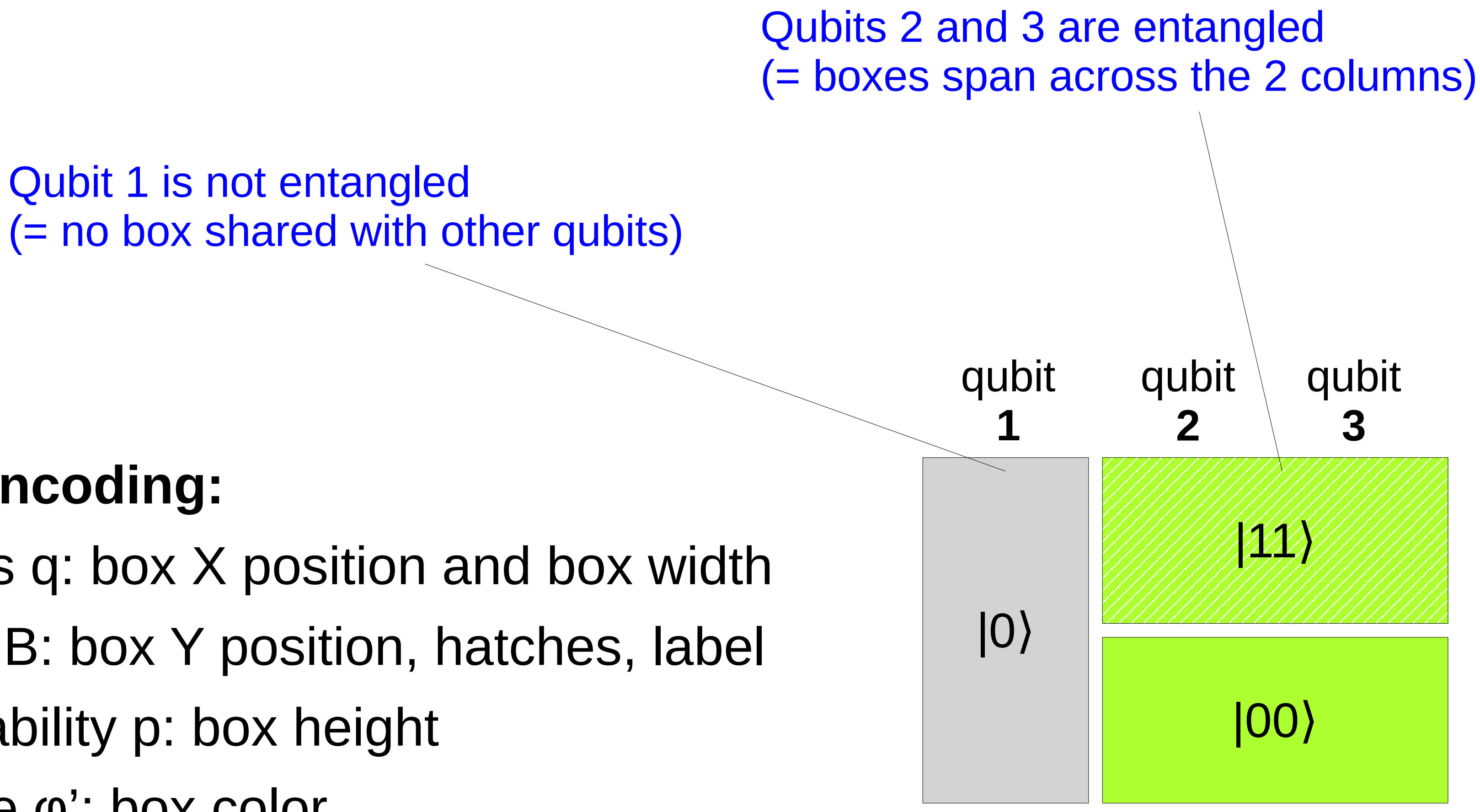
{ (q, B, p, φ’) }
( {1}, |0⟩, 100%, 0 )
( {2, 3}, |00⟩, 50%, 0 )
( {2, 3}, |11⟩, 50%, 0 )




Opposite color => opposite phase (i.e. Z gate)

# Visual representation

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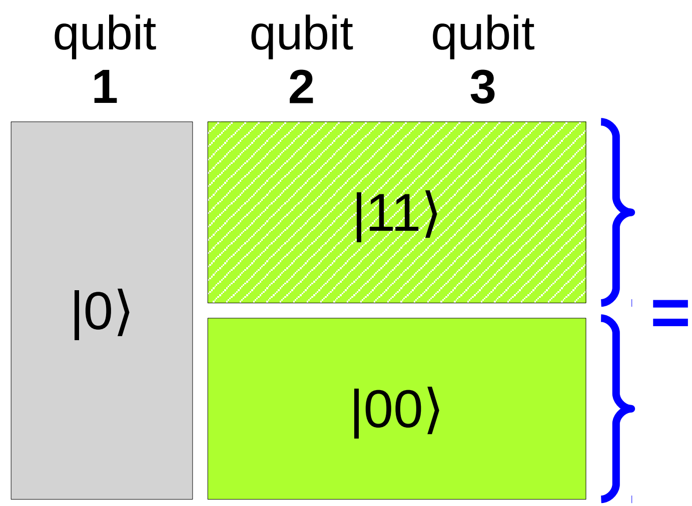


Same probability of measuring |00⟩ and |11⟩
(= same box height)

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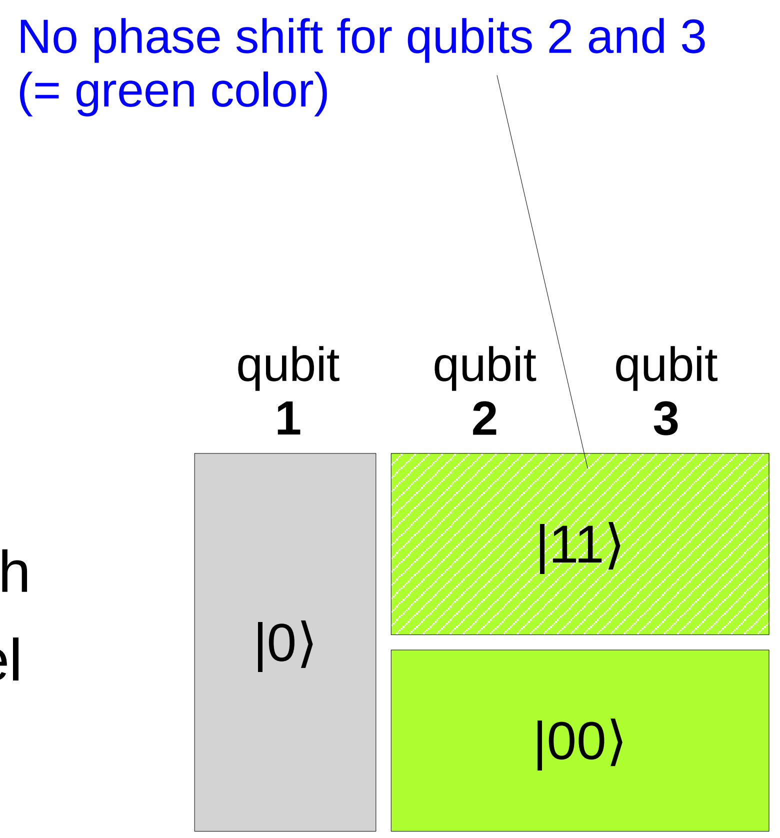
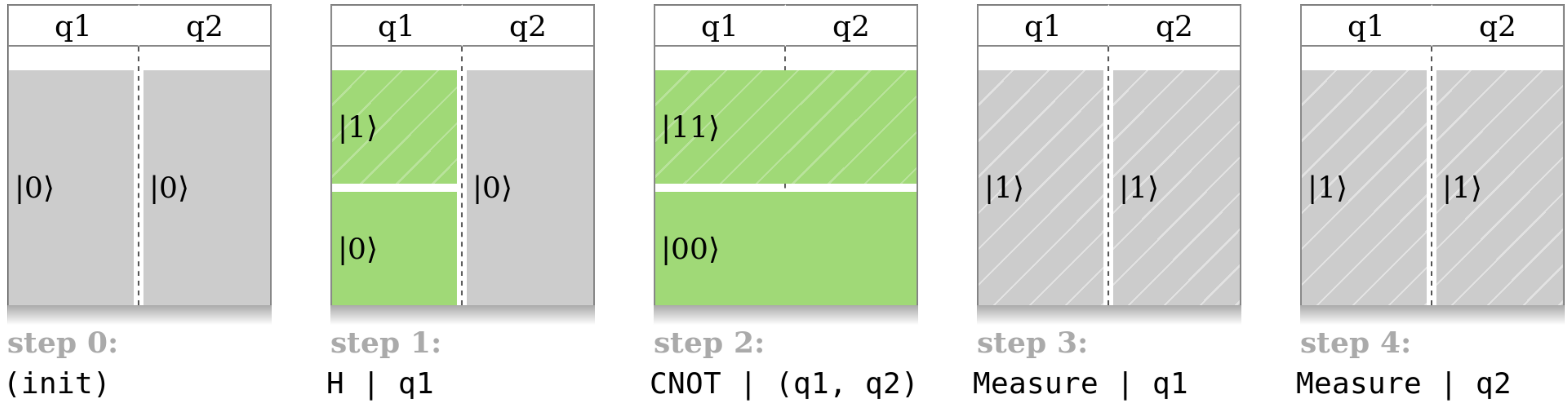


# Implementation

- **Python 3**
- **ProjetQ**
  - Python module for quantum computing
  - Compiles quantum circuits for various hardware
  - Can also *simulate* a quantum computer on a classical hardware
    - In simulation mode, one can access the inner states of the qubits (which is not possible with quantum hardware)

# Application to Bell pair

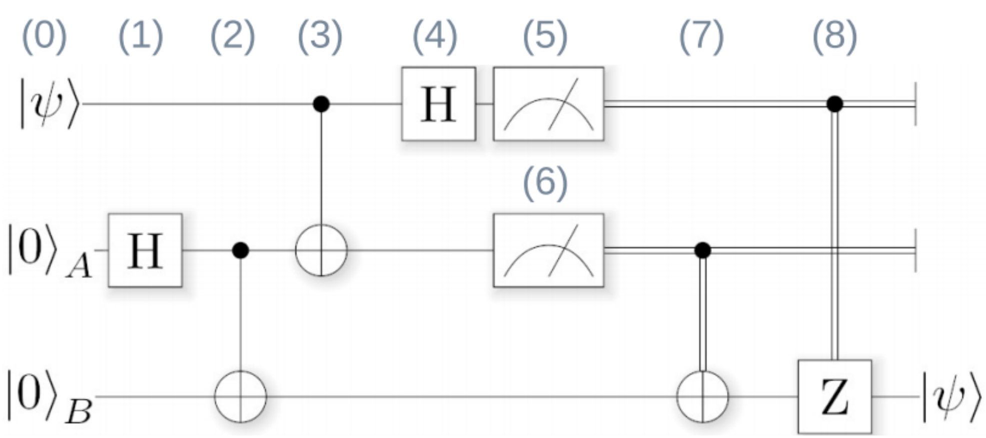
- **Example on the Bell pair**
  - Bell pair : two qubits with maximal level of entanglement
    - Hadamard gate (H)
    - Conditional Not gate (CNOT)
    - Then measurement
  - One set of rainbow boxes for each step of the algorithm

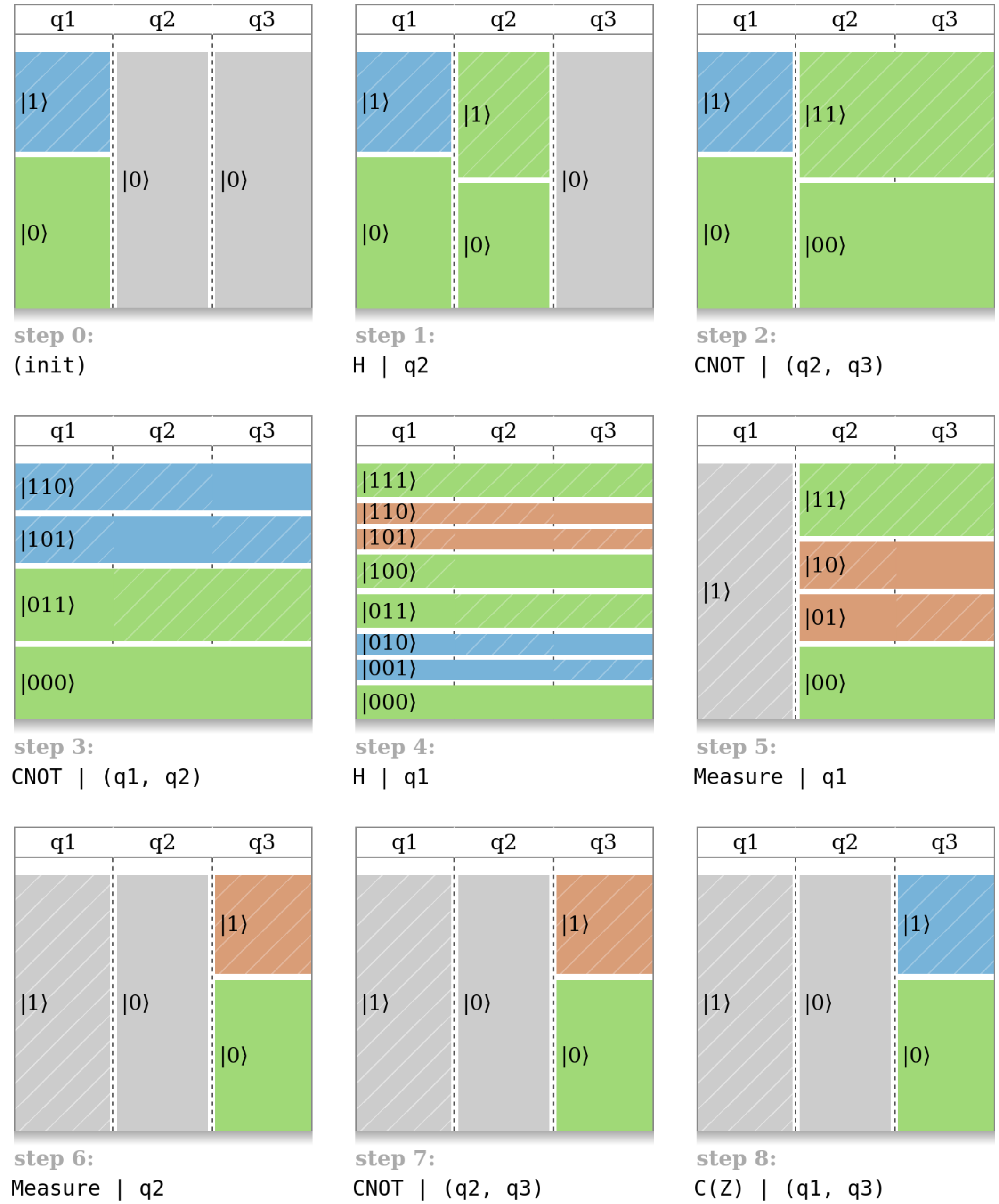


# Application to quantum teleportation

**Objective:** “teleport” the value of q1

- Step 1 and 2 creates a Bell pair



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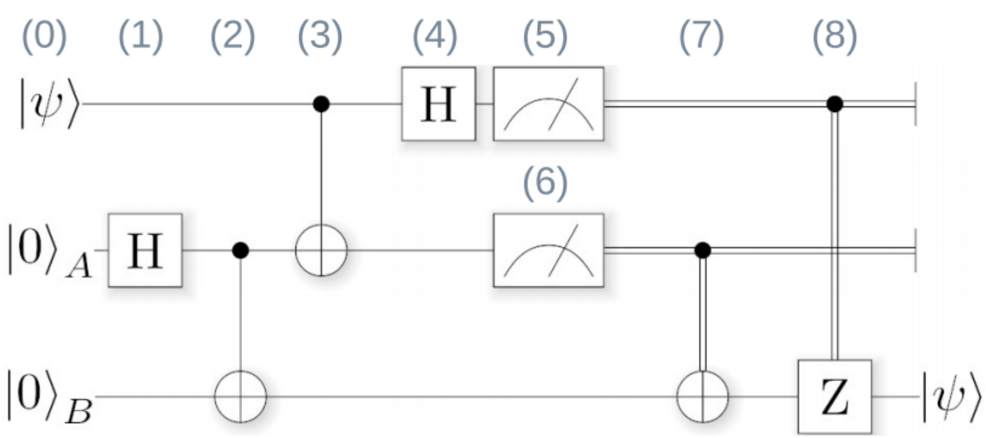


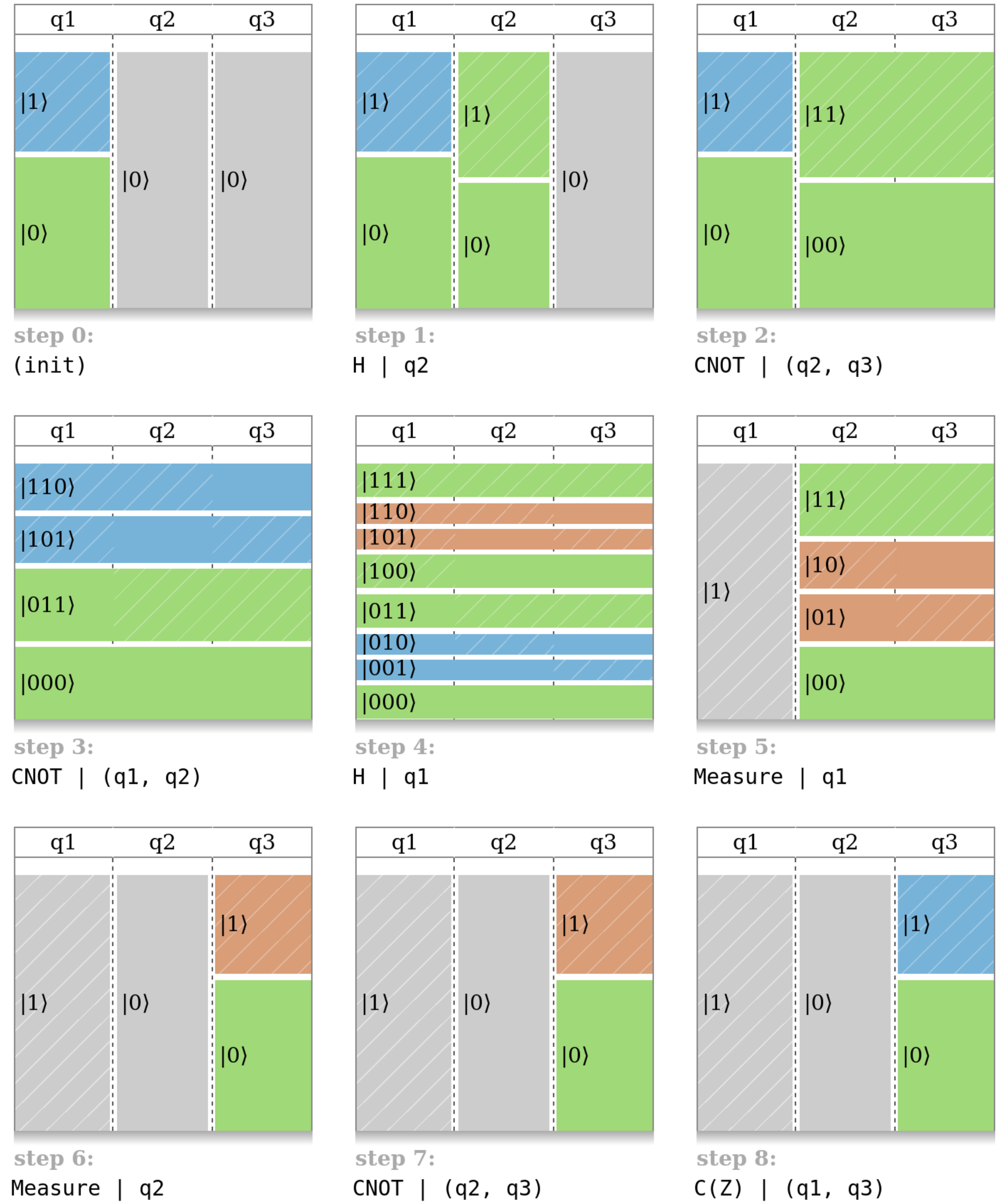

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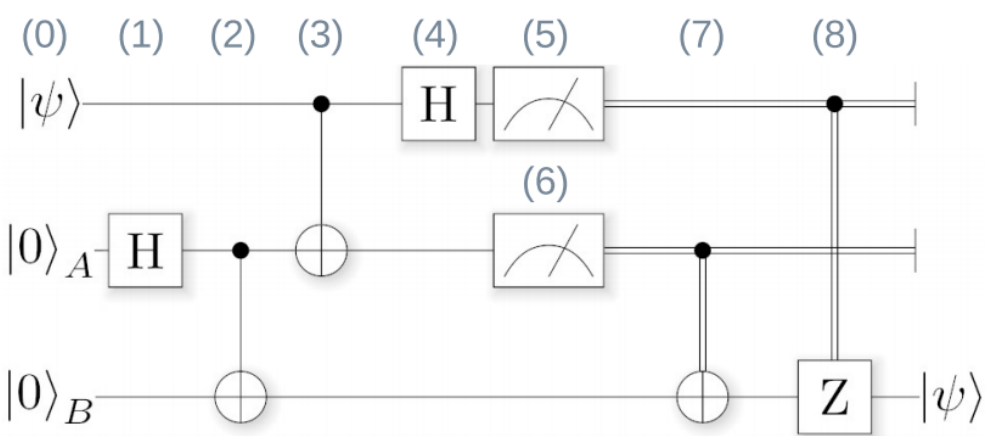
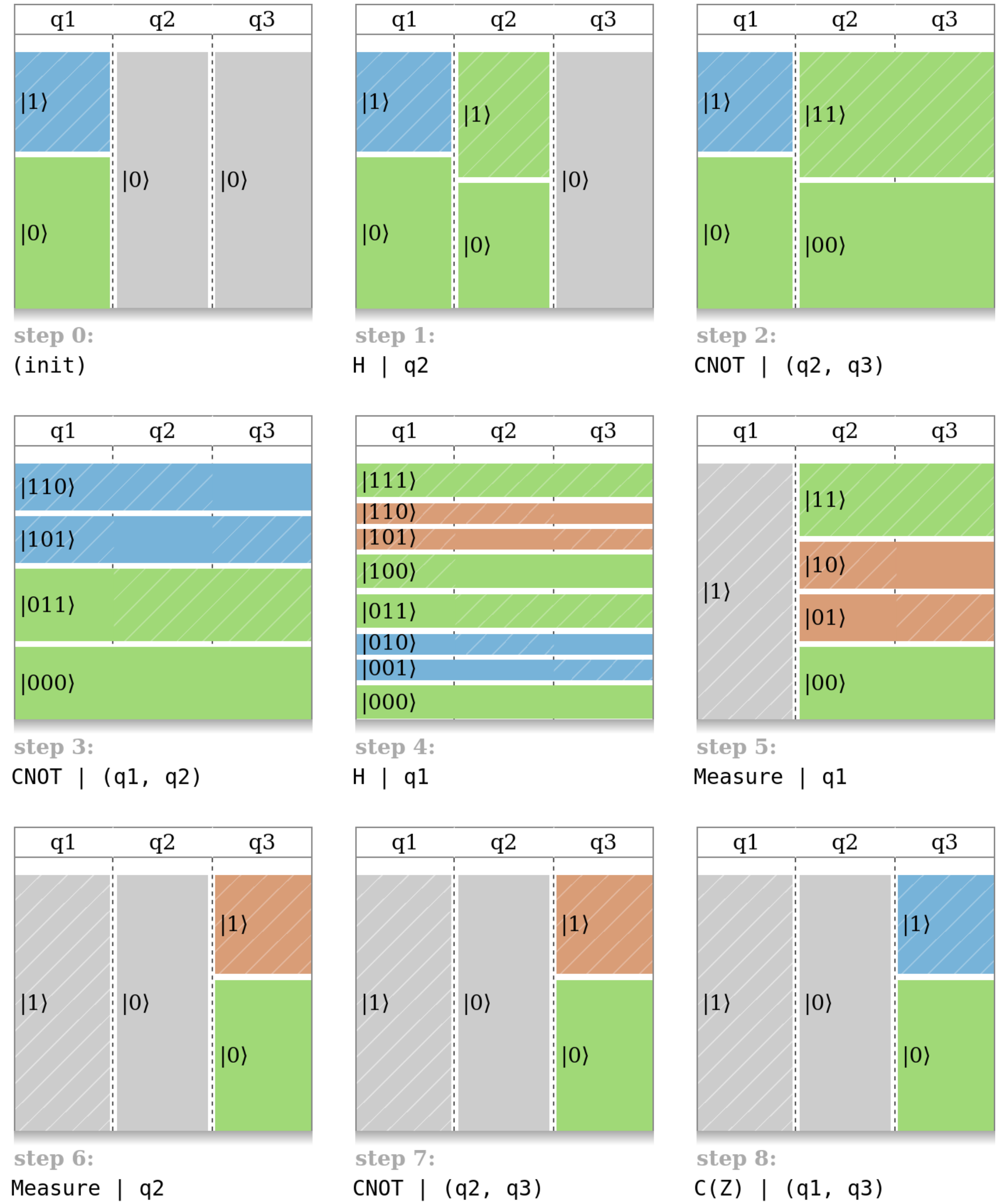
- Step 4 apply Hadamard gate
  => probability of measuring 0 or 1
  is now 50% for all qubits
  but the initial value is not lost!



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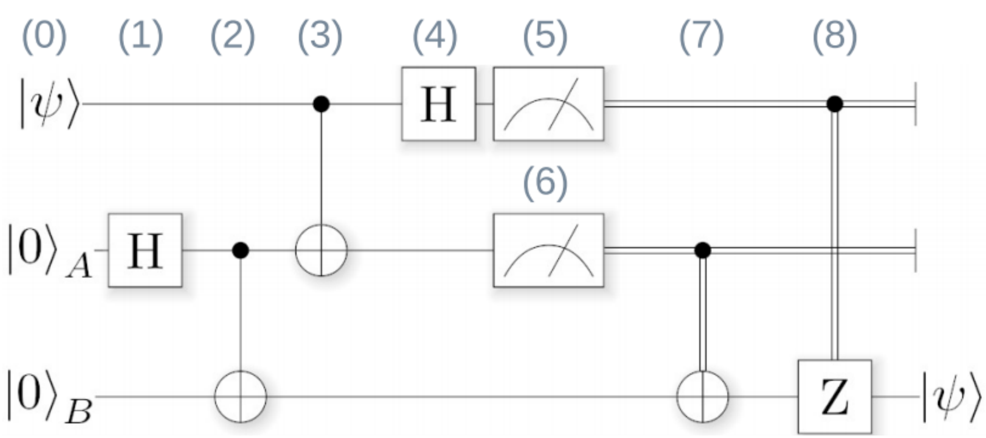


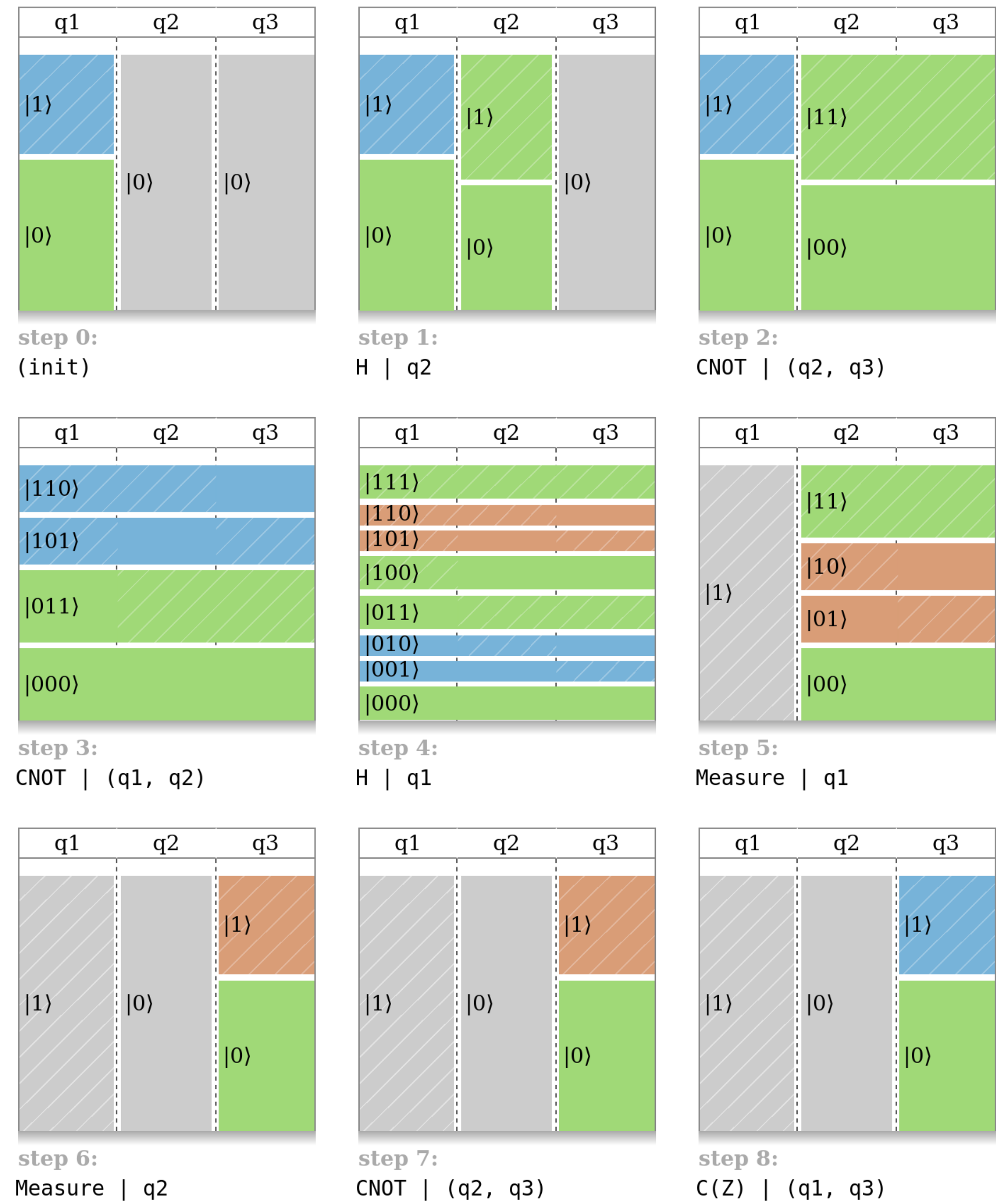

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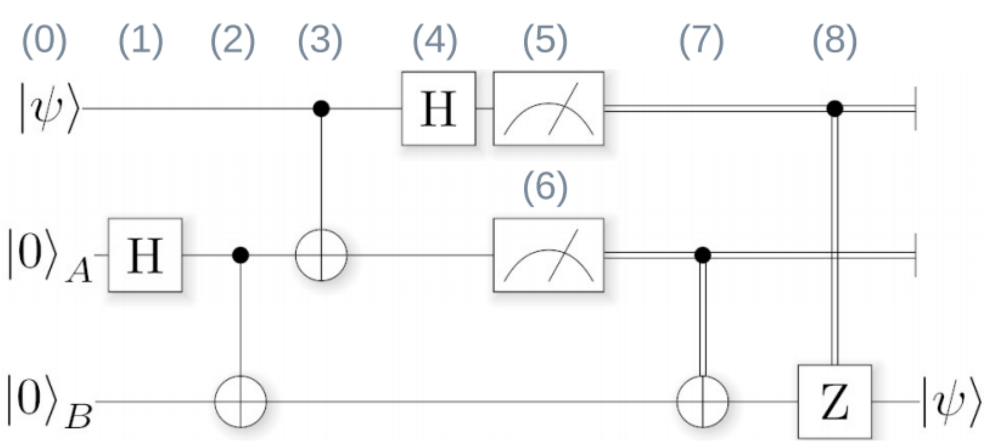
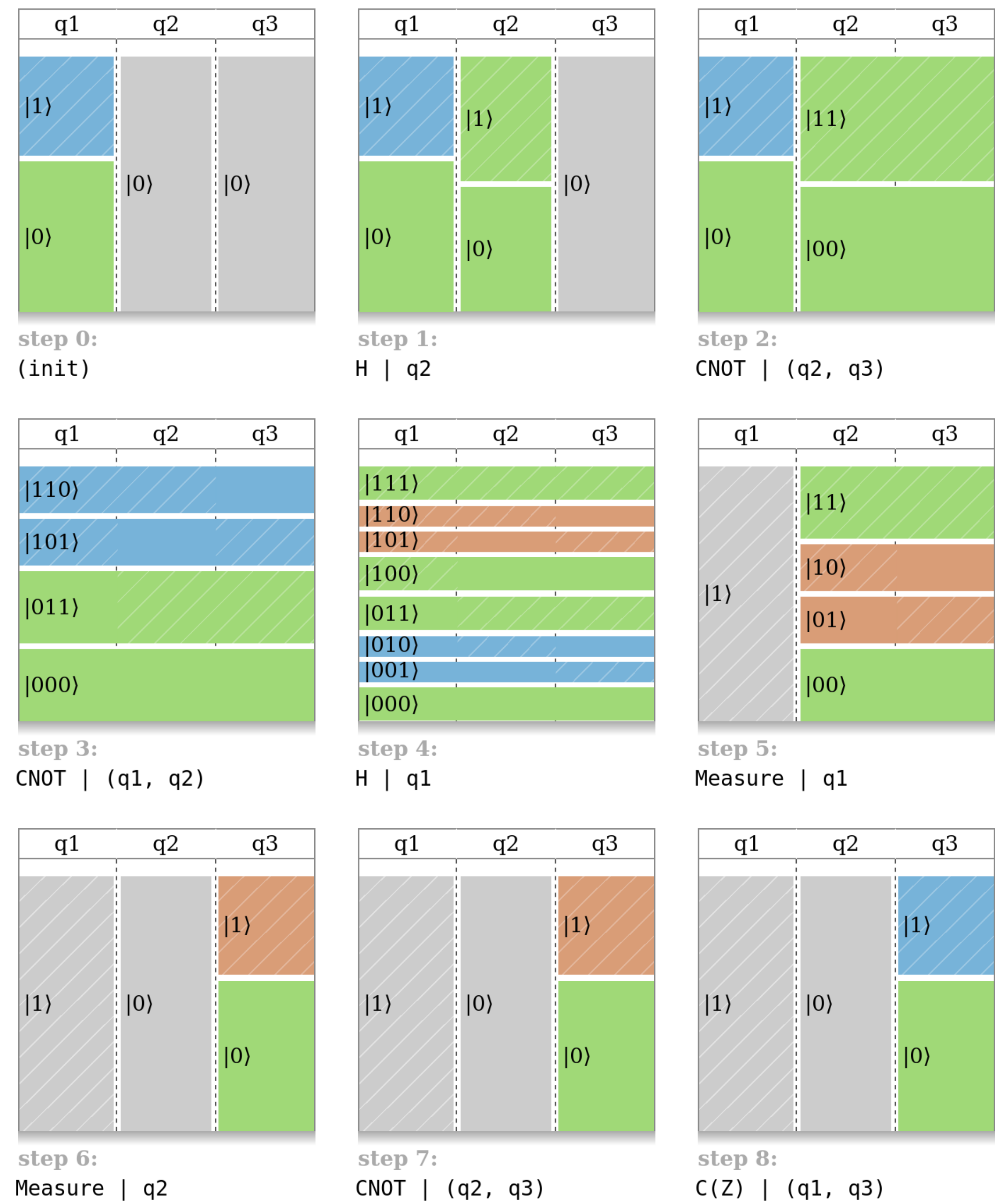
- Step 7 and 8 rebuild the value in q3,
  according to the values measured



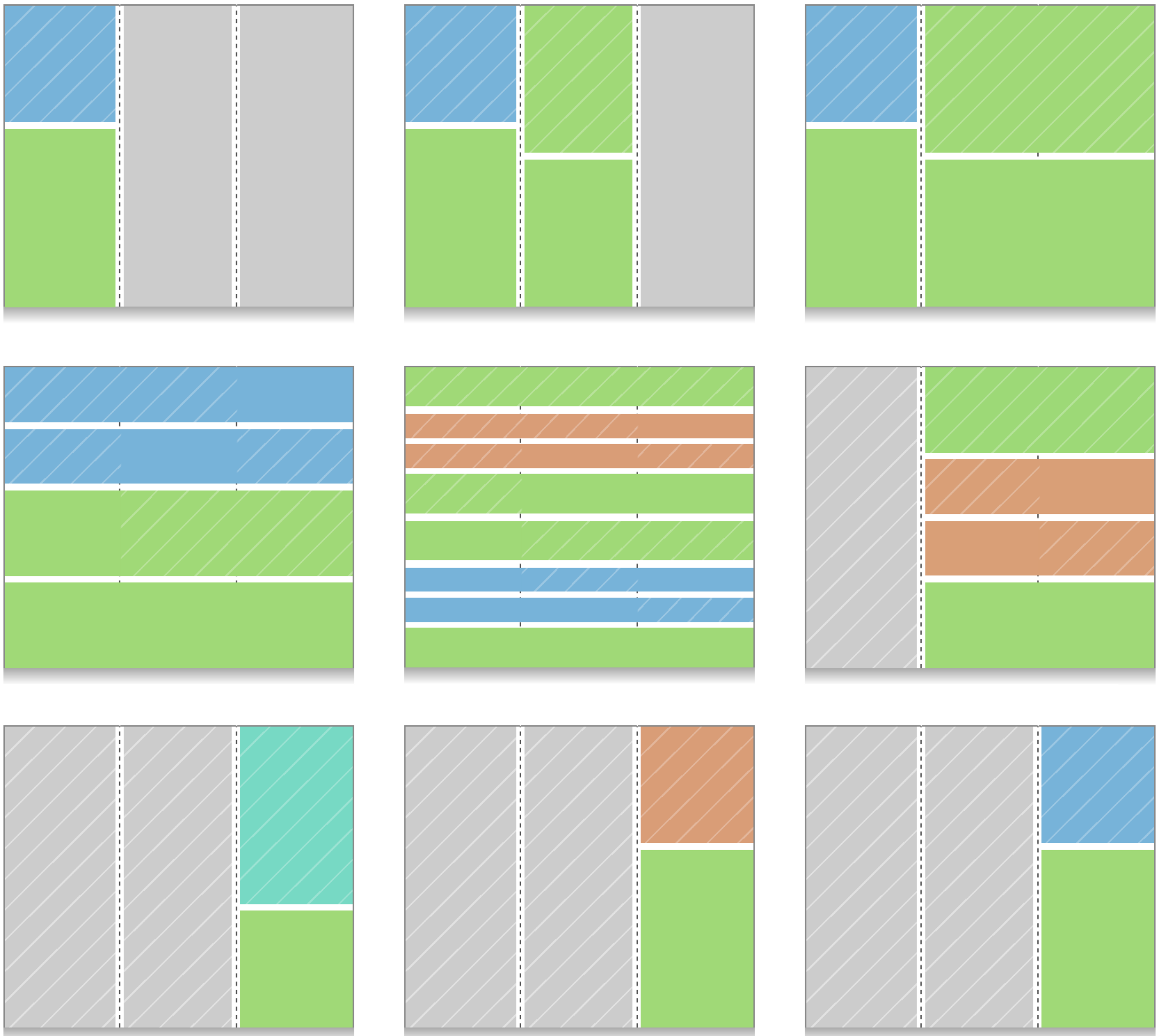
# Discussion

- **Most visual approach to quantum computing relies on Bloch sphere or complex plane (CP)**
  - We showed that set visualization is another possibility
  - The proposed state visualization is unique
    - Different visual representations imply different states
- **Interesting for teaching quantum computing**
  - Quantum theory is known to be unintuitive
  - Allows an empirical and experimental “trial and error” approach to quantum computing
    - *e.g.* testing quantum teleportation on different initial states, or testing modified algorithms (“what about swapping step 7 and 8?”)

# Perspectives

- Use and evaluation in education
- Integration in ProjectQ
- Extension to other quantum computing paradigm, beyond quantum circuits

# Questions?



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